Midterm Exam

Statistical Physics

Friday December 12, 2014
14:00-16:00

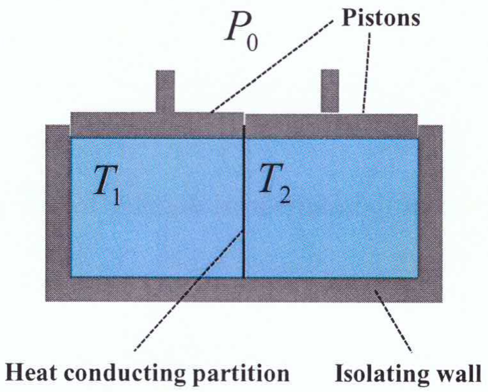
Read these instructions carefully before making the exam!

- **Write your name and student number on *every* sheet.**
- ***Make sure to write readable for other people than yourself. Points will NOT be given for answers in illegible writing.***
- ***Language*; your answers have to be in English.**
- **Use a *separate* sheet for each problem.**
- **Use of a (graphing) calculator is allowed.**
- **This exam consists of 3 problems.**
- **The weight of the problems is Problem 1 (P1=27 pts); Problem 2 (P2=27 pts); Problem 3 (P3=36 pts). Weights of the various subproblems are indicated at the beginning of each problem.**
- **The grade of the midterm exam is calculated as (P1+P2+P3+10)/10.**
- **For all problems you have to write down your arguments and the intermediate steps in your calculation, *else the answer will be considered as incomplete and points will be deducted.***

PROBLEM 1

Score: a+b+c =9+9+9=27

A mass M of liquid at a temperature T_1 is in thermal contact (through a fixed heat conducting partition) with an equal mass of the same liquid at a temperature T_2 (see figure). The system is thermally insulated but the liquids are maintained at some constant pressure P_0. After some time the liquid in both compartments obtains the same temperature T_f. The heat capacity (per mass) at constant pressure C_P of the liquid may be assumed independent of temperature.



a) Calculate T_f.
b) Show that the total entropy change ΔS_{tot} of the system is:

$$\Delta S_{tot} = 2MC_P \ln\left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right)$$

c) Proof that $\Delta S_{tot} \geq 0$

PROBLEM 2

Score: a+b+c =9+9+9=27

A crystal in equilibrium with a heat bath at temperature T contains N similar, statistically independent, defects. Each defect has five possible states $r = 1, 2, 3, 4, 5$. The energy of these states are: $E_1 = E_2 = 0$ and $E_3 = E_4 = E_5 = \varepsilon$.

a) Give the partition function of the N defects.
b) Calculate the defects contribution to the entropy of the crystal.
c) Show that if $kT \gg \varepsilon$ then the mean energy of the N defects is given by $\bar{E} = \frac{3}{5} N\varepsilon$.

PROBLEM 3

Score: a+b+c+d =8+8+8+12=36

A harmonic oscillator with energy levels given by $\varepsilon_j = \hbar\omega(j + \frac{1}{2})$ is in equilibrium with a heat bath at temperature T. ω is the angular frequency of the oscillator.

a) Proof that the mean energy $\overline{\varepsilon}$ of this oscillator is given by: $\overline{\varepsilon} = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\beta\hbar\omega}-1}$

Now consider a 2-dimensional square crystal that consist of N atoms. Both sides of the crystal have length L. Assume that the crystal can be described as a system of 2N coupled oscillators.

b) Use Debye's theory to show that the number of angular frequencies between ω and $\omega + d\omega$ is given by:

$$f(\omega)d\omega = \frac{L^2\omega}{\pi v_0^2}d\omega$$

In this expression v_0 is the velocity of the transverse and longitudinal waves which are assumed to be equal. Assume that the waves have one transversal en one longitudinal mode.

c) Proof that the Debye frequency ω_D for this 2-dimensional crystal is:

$$\omega_D = \sqrt{4\pi N}\frac{v_0}{L}$$

d) Give an expression for the heat capacity C_V of this 2-dimensional crystal and show that in case $T \to 0$ then C_V decreases like T^2.

Physical contants:

Avogadro's number:	N_0= 6.02 x 10^{23} mol^{-1}
Planck's constant:	h = 6.626 x 10^{-34} Js
	$\hbar = \frac{h}{2\pi}$ = 1.055 x 10^{-34} Js
Boltzmann's constant:	k = 1.381 x 10^{-23} J K^{-1}
Gas constant:	R = 8.31 J mol^{-1} K^{-1}
Speed of light:	c = 3 x 10^8 m s^{-1}
Electron rest mass:	m_e = 9.11 x 10^{-31} kg
Proton rest mass:	m_p = 1.67 x 10^{-27} kg
Charge of the eletron:	e = 1.60 x 10^{-19} C
Bohr magneton:	$\mu_B = \frac{e\hbar}{2m_e}$ = 9.27 x 10^{-24} A m^2
Permeability of vacuum:	$\mu_0 = 4\pi$ x 10^{-7} N A^{-2}
Molar volume at STP:	22.4 litre

Integrals:

n	$\int_0^\infty dx\, x^n e^{-ax^2}$ $(a > 0)$	$\int_0^\infty \frac{x^n dx}{e^x - 1}$	$\int_0^\infty \frac{x^n dx}{e^x + 1}$	$\int_0^\infty \frac{x^n e^x}{(e^x - 1)^2}$	$\int_0^\infty x^n \ln(1 - e^{-x}) dx$
0	$\frac{1}{2}\sqrt{\left(\frac{\pi}{a}\right)}$	diverges	ln 2	diverges	$-\frac{\pi^2}{6}$
1/2	$\frac{0.6127}{a^{3/4}}$	$2.612\,\frac{\sqrt{\pi}}{2}$	0.6781	diverges	$-1.341\,\frac{\sqrt{\pi}}{2}$
1	$\frac{1}{2a}$	$\frac{\pi^2}{6}$	$\frac{\pi^2}{12}$	diverges	-1.202
3/2	$\frac{0.4532}{a^{5/4}}$	$1.341\,\frac{3\sqrt{\pi}}{4}$	1.153		$-1.127\,\frac{3\sqrt{\pi}}{4}$
2	$\frac{1}{4a}\sqrt{\frac{\pi}{a}}$	2.404	1.803	$\frac{\pi^2}{3}$	$-\frac{\pi^4}{45}$
5/2	$\frac{1.662}{a^{7/4}}$	$1.127\,\frac{15\sqrt{\pi}}{8}$	3.083		-3.505
3	$\frac{1}{2a^2}$	$\frac{\pi^4}{15}$	$\frac{7\pi^4}{120}$	7.212	-6.221
7/2	$\frac{0.5665}{a^{9/4}}$	12.268	11.184		
4	$\frac{3\sqrt{\pi}}{8a^{5/2}}$	24.886	23.331	$\frac{4\pi^4}{15}$	